

Cambridge International AS & A Level

CANDIDATE
NAME

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

1 Solve the inequality $|2x - 1| > 3|x + 2|$.

[4]

[illegible]

2 Find the exact value of $\int_0^1 (2-x)e^{-2x} dx$.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 3 (a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x .

[2]

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- (b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places.

[4]

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4 The equation of a curve is $y = x \tan^{-1}\left(\frac{1}{2}x\right)$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) The tangent to the curve at the point where $x = 2$ meets the y -axis at the point with coordinates $(0, p)$.

Find p . [3]

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5 By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 90^\circ$.

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a)** By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

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- (b)** Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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- (c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 Let $f(x) = \frac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions.

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(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}. \quad [2]$$

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(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

[illegible]

- 8** Relative to the origin O , the points A , B and D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point C is such that $ABCD$ is a parallelogram.

- (a) Find the position vector of C and verify that the parallelogram is not a rhombus. [5]

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(b) Find angle BAD , giving your answer in degrees.

[3]

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(c) Find the area of the parallelogram correct to 3 significant figures.

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- 9 (a) The complex numbers u and w are such that

$$u - w = 2i \quad \text{and} \quad uw = 6.$$

Find u and w , giving your answers in the form $x + iy$, where x and y are real and exact. [5]

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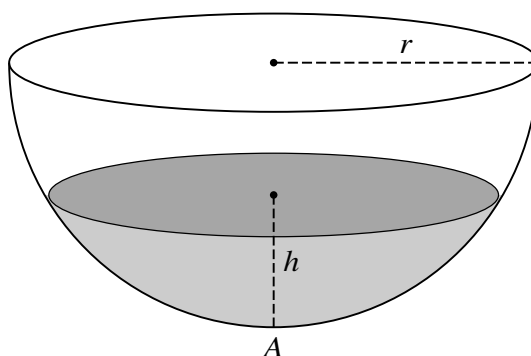
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- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$



The volume of water in the tank is V when the depth is h . It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

[4]

[illegible]

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[illegible]

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